Hypothesis Testing for Proportions

Sections 9.1 - 9.2

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Outline

The Parameters

- Any question about a population must first be described in terms of a population parameter.
- We will work with only two parameters:
  - The population mean μ.
  - The population proportion p.

The Two Basic Questions

The question about that parameter will fall into one of two categories.

- Estimation
  - What is the value of the parameter?
- Hypothesis testing
  - Does the evidence support or refute a claim about the value of the parameter?

Example

If we want to learn about voters' preferences...

- What parameter do we use?
- Do we estimate the parameter?
- Or do we test a hypothesis?

Example

If we want to learn about the effectiveness of a new drug...

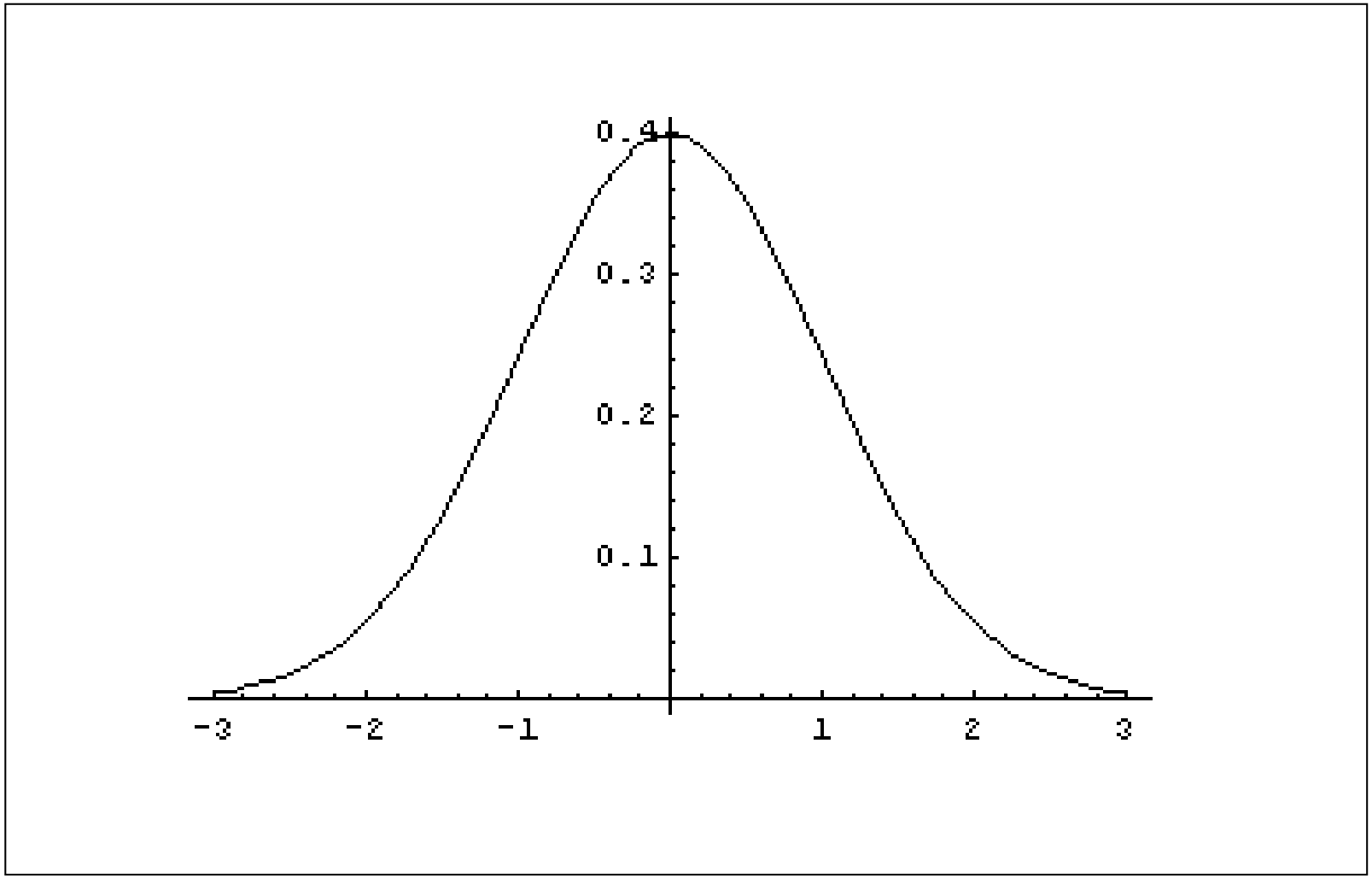
- What parameter do we use?
- Do we estimate the parameter?
- Or do we test a hypothesis?

Example

If we want to find out whether a newborn child is more likely to be male than female...

- What parameter do we use?
- Do we estimate the parameter?
- Or do we test a hypothesis?

The *p*-Value Approach



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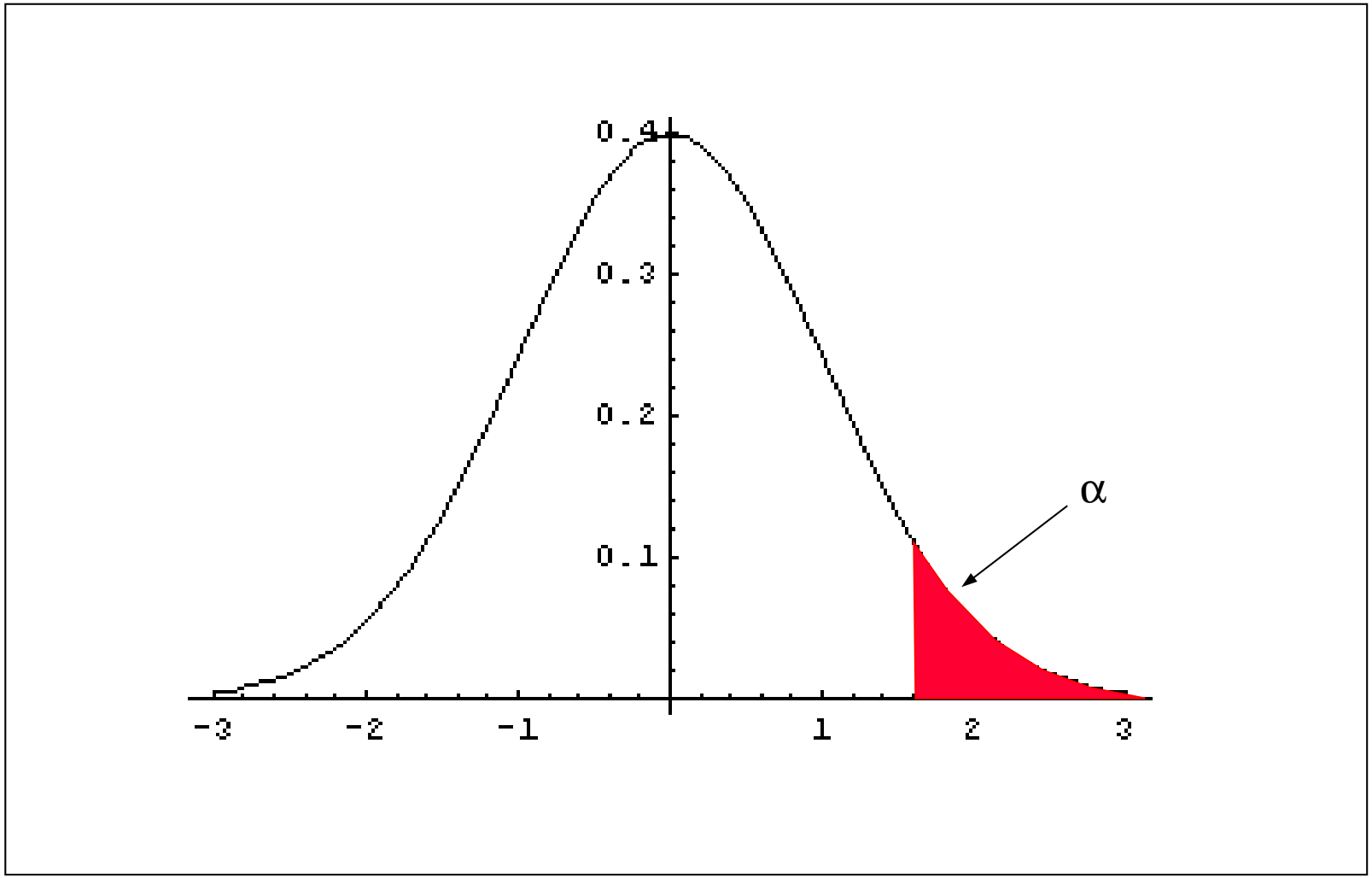

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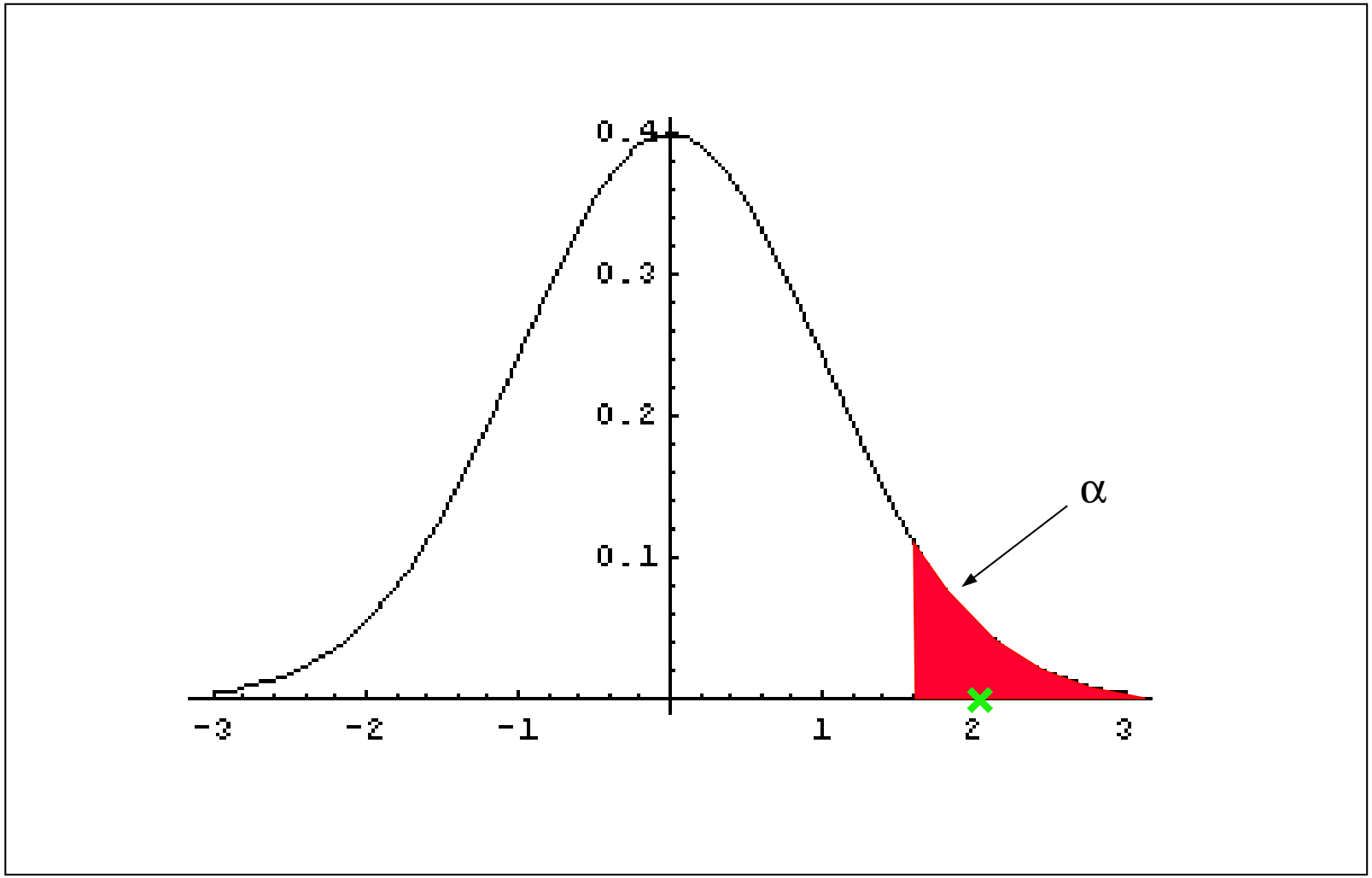

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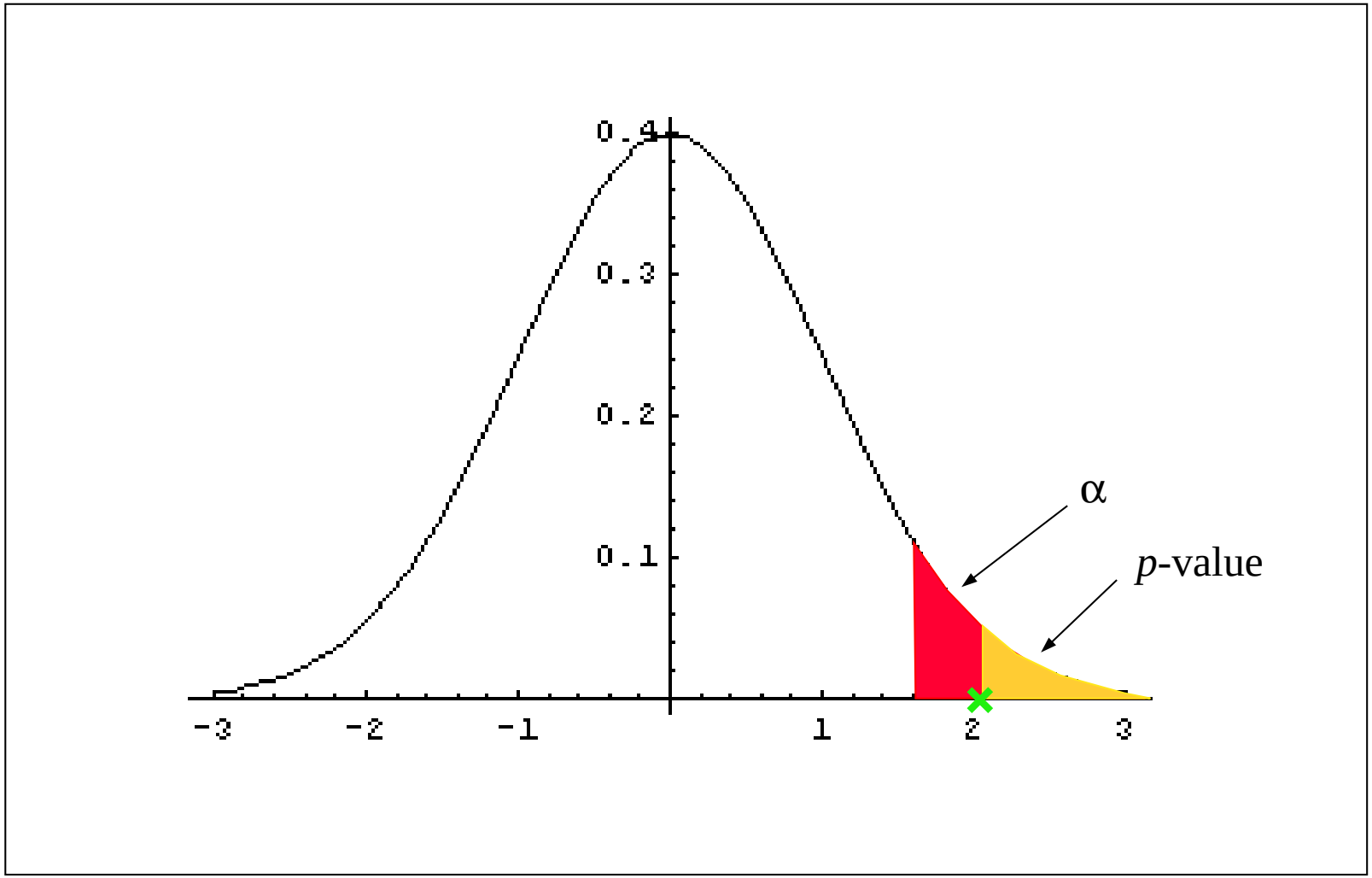

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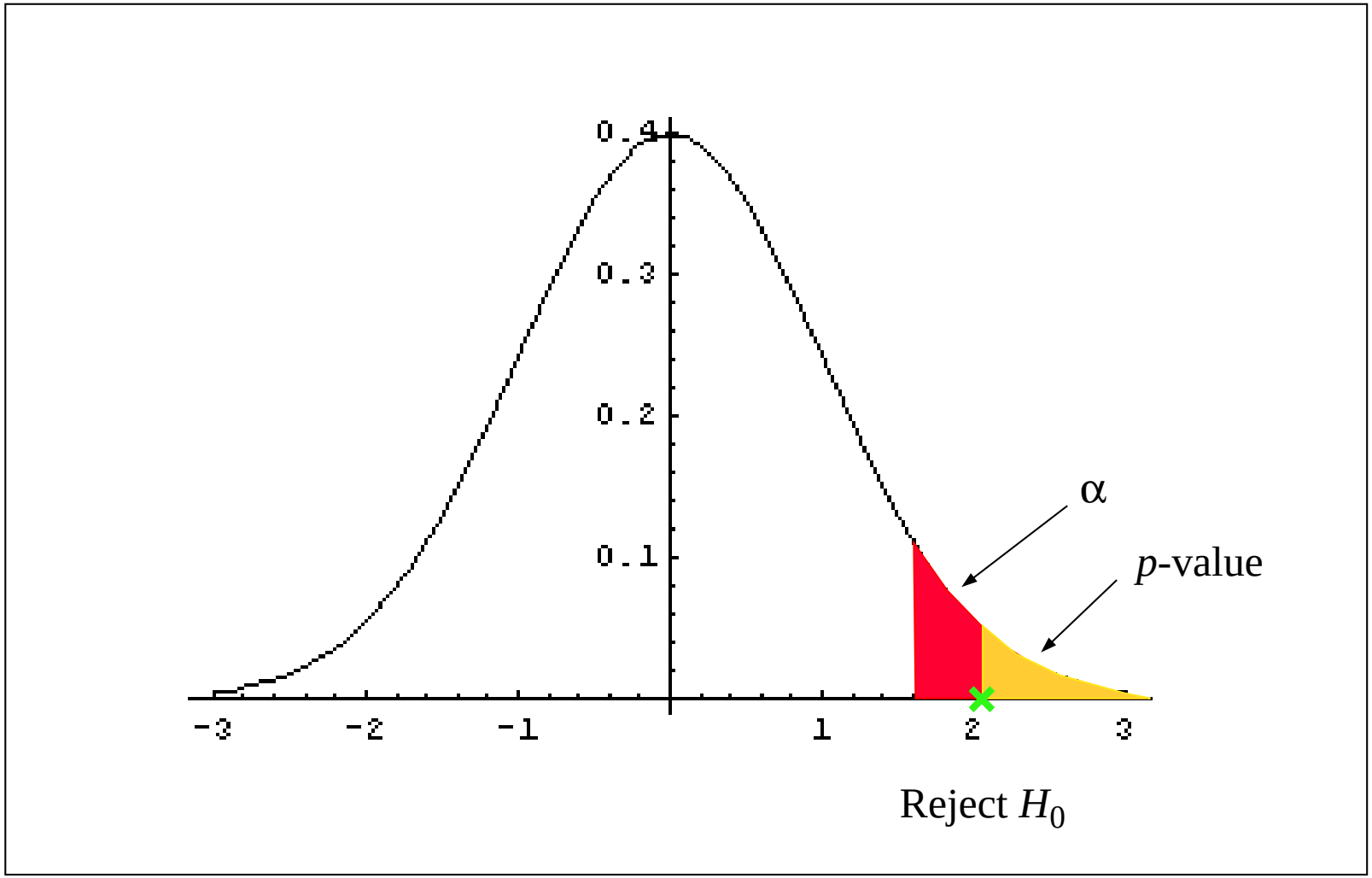

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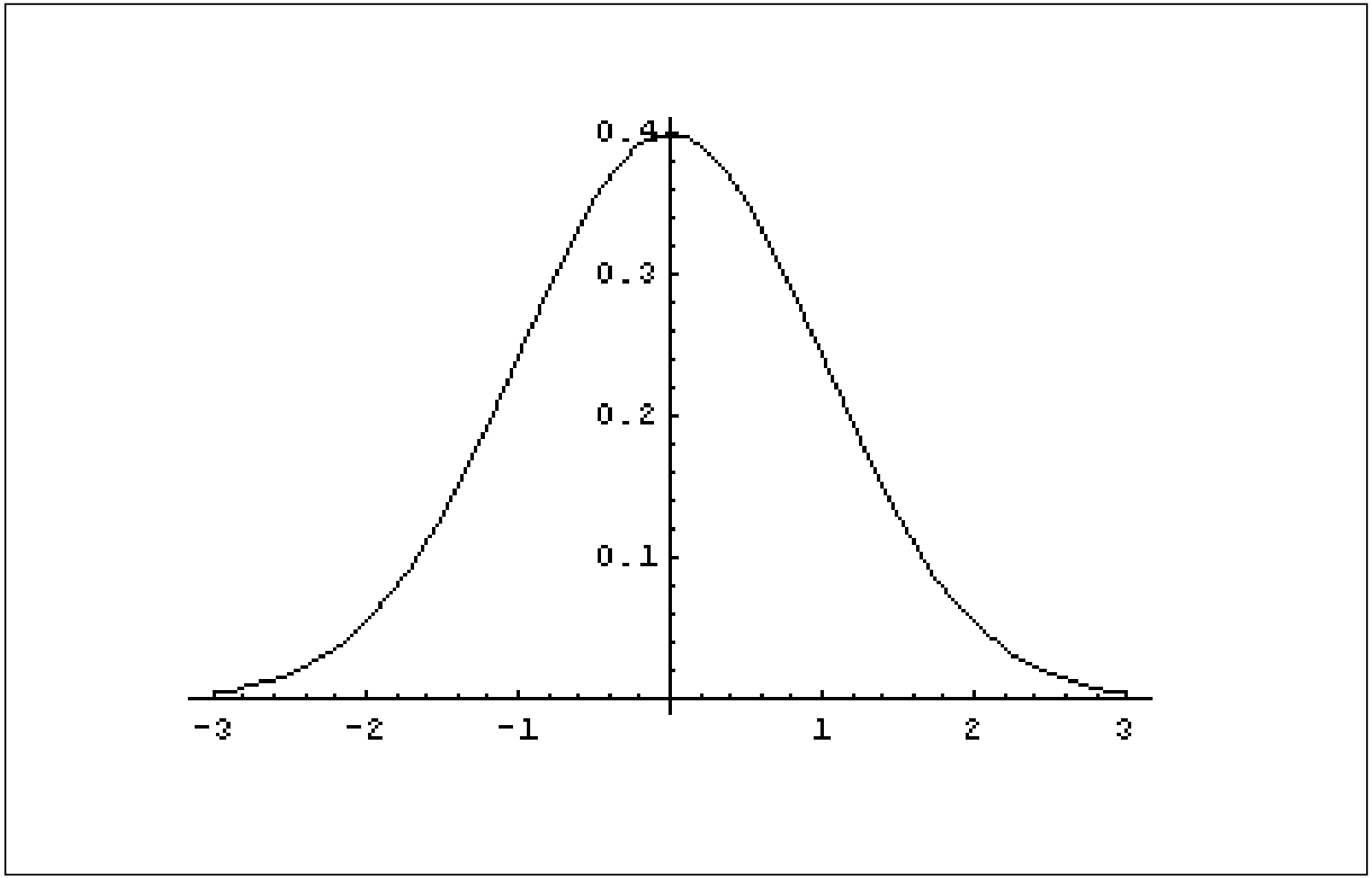

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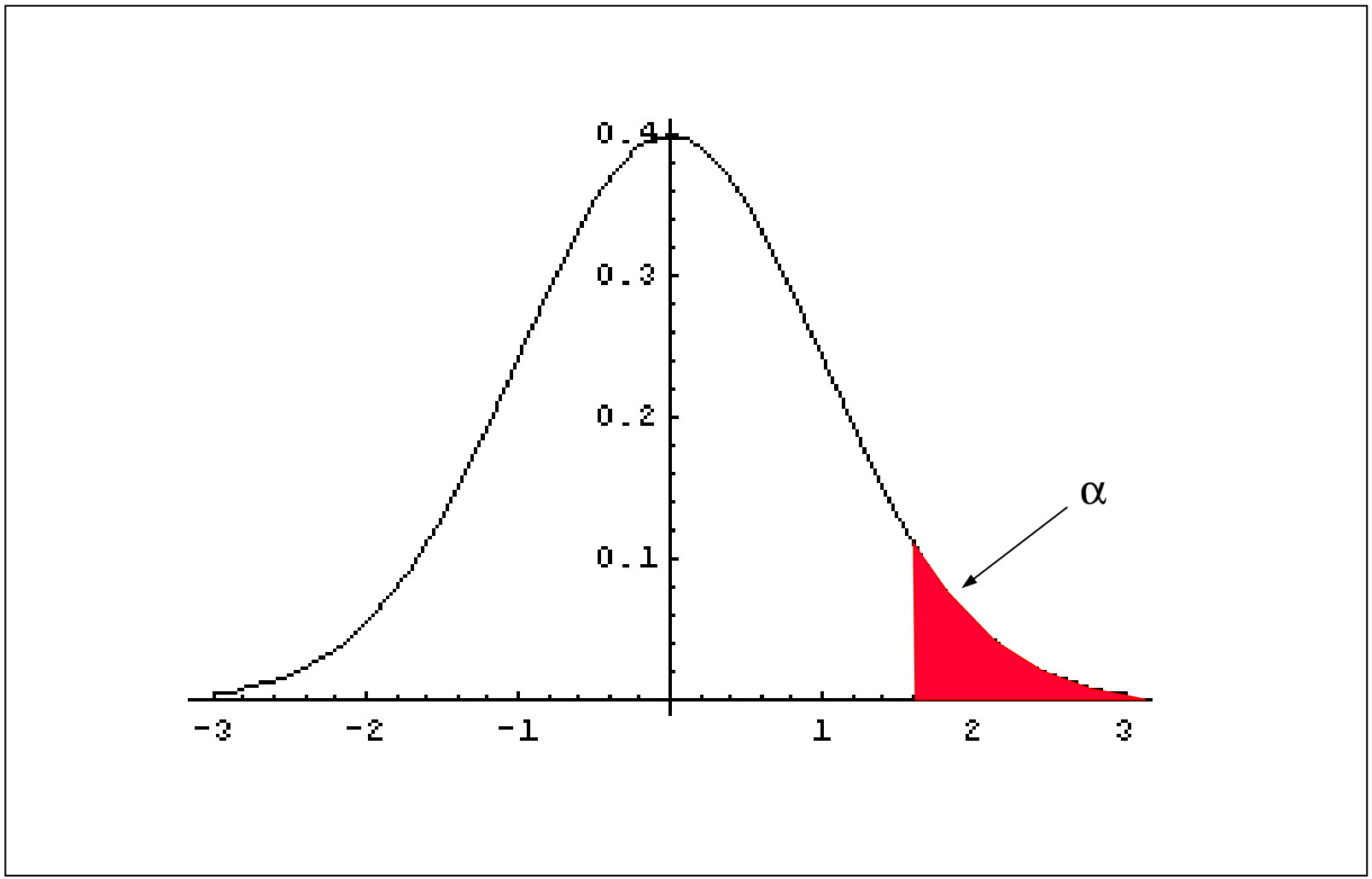

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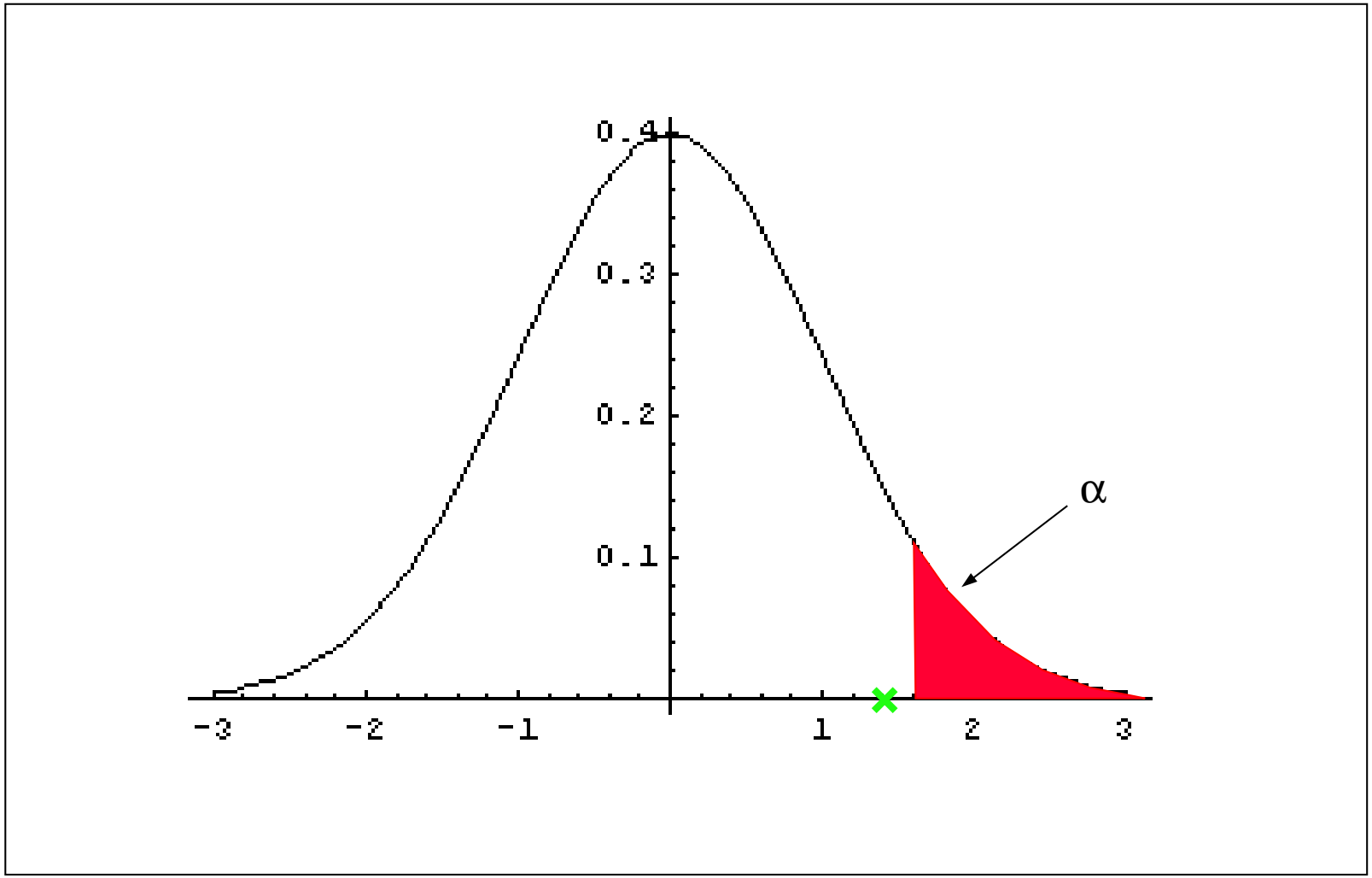

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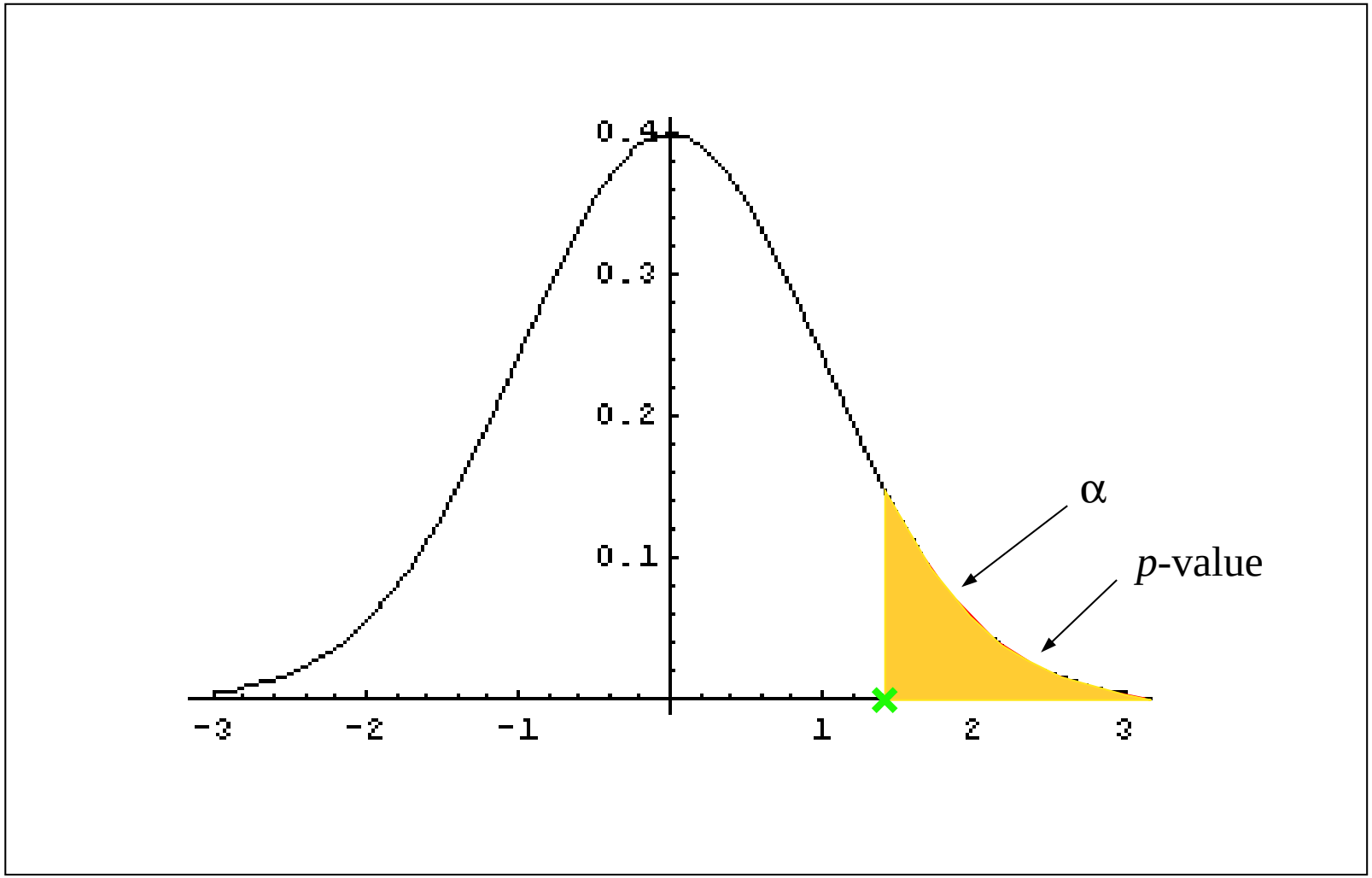

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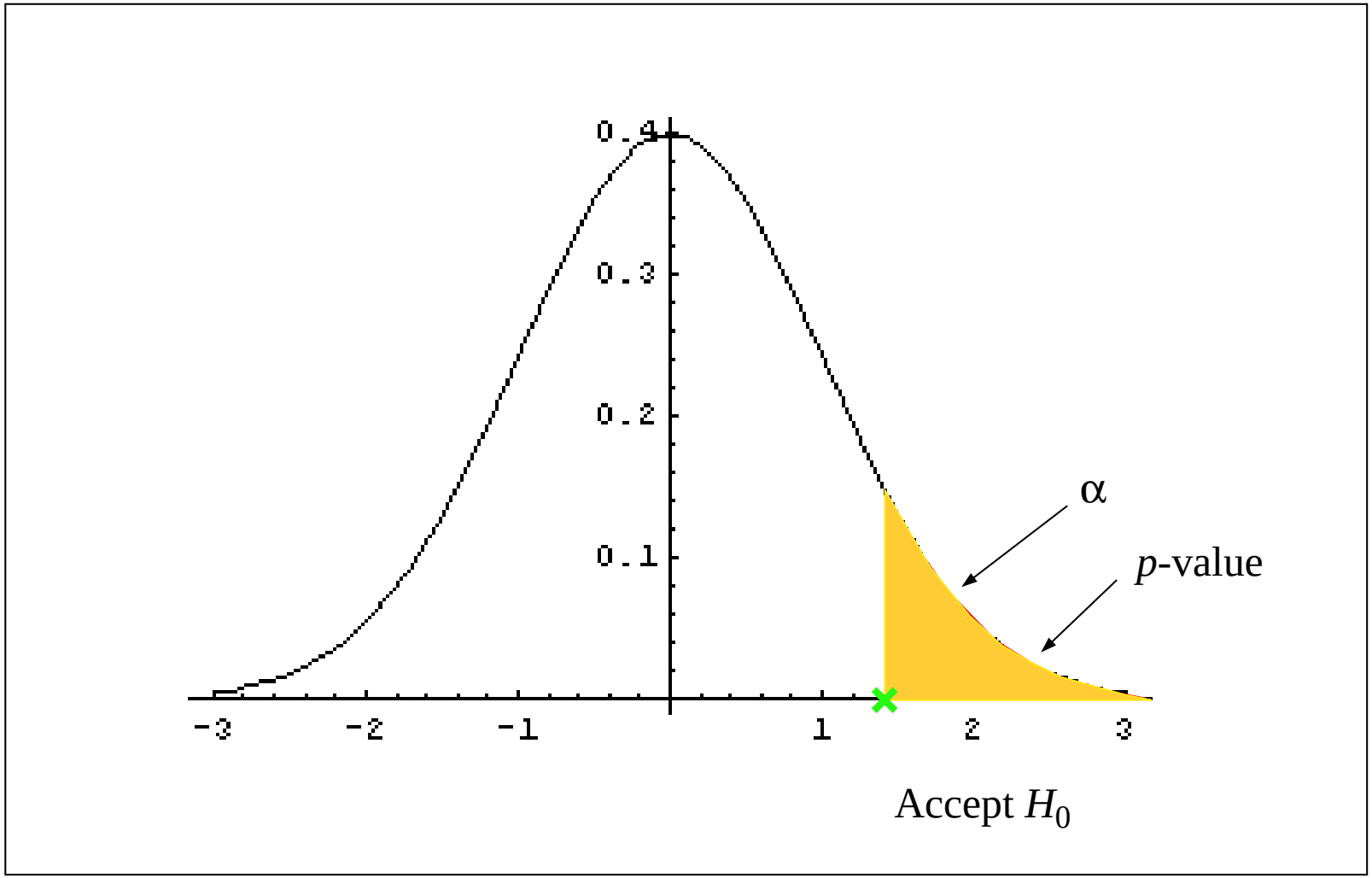

The Steps of Testing a Hypothesis

p-Value Approach

The seven steps:

1. State the null and alternative hypotheses.
2. State the significance level.
3. State the formula for the test statistic.
4. Compute the value of the test statistic.
5. Compute the p-value.
6. Make a decision.
7. State the conclusion.

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An Illustrative Example

We will use the following example to illustrate the seven steps.

- Are male births more common than female births?
- Suppose a random sample of 1000 live births shows that 520 are males and 480 are females.
- Test the hypothesis that male births are more common than female births, at the 5% level of significance.

The Parameter

- Select the appropriate parameter and describe what it represents.

Let $p =$ proportion of births that are males.

The Null Hypothesis

- The null hypothesis should state a hypothetical value p_0 for the population proportion.

$$H_0 : p = p_0.$$

The Alternative Hypothesis

- The alternative hypothesis must contradict the null hypothesis in one of three ways:
  - $H_1 : p < p_0$. (Direction of extreme is left.)
  - $H_1 : p > p_0$. (Direction of extreme is right.)
  - $H_1 : p \neq p_0$. (Direction of extreme is left and right.)

The Example

- In our example, the null and alternative hypotheses are

$$H_0 : p = 0.50.$$
$$H_1 : p > 0.50.$$

The Level of Significance

- Specify the level of significance α.
- In our example,

$$\alpha = 0.05$$

The Test Statistic

- State the formula to be used for the test statistic.
- Be sure to include the “name" of the statistic (e.g., Z).
- In our example, we need to figure out what the test statistic is.

The Example

- The statistic $\hat{p}$ has a normal distribution with

$$\mu_{\hat{p}} = p$$

and

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}.$$

- Therefore, if we use the z-score, we have the test statistic

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.$$

The Value of the Test Statistic

- To compute the value of the test statistic, we substitute the values of $\hat{p}$, p_0, and n.
- In our example, $\hat{p} = \frac{520}{1000} = 0.52$, $p_0 = 0.50$, and $n = 1000$.

$$\begin{aligned} Z &= \frac{0.52 - 0.50}{\sqrt{\frac{(0.50)(1-0.50)}{1000}}} \\ &= \frac{0.02}{0.01581} \\ &= 1.265 \end{aligned}$$

The *p*-Value

- The *p*-value is the probability that Z would be at least as extreme as the value that we computed.
- To find the *p*-value, use the `normalcdf` function on the TI-83.
- Pay attention to the direction of extreme.
- If the test is two-tailed, then we should double the value given by the calculator.

The Example

- In our example, $Z = 1.265$.
- The direction of extreme is to the right.
- Calculate

$$p\text{-value} = \texttt{normalcdf(1.265,E99)} = 0.1029.$$

The Decision

- The decision states whether to accept or reject the null hypothesis.
- In our example, the *p*-value is greater than α, so our decision is:

Do not reject the null hypothesis.

The Conclusion

- The conclusion restates the decision in the language of the original problem.
- In our example, the conclusion is

  The proportion of male births is equal to 50%.

Summary

The following is a minimal write-up of the problem. You may want to show more work for partial credit.

1. Let p = the proportion of male births.
   $$H_0 : p = 0.50$$
   $$H_1 : p > 0.50$$
2. $\alpha = 0.05$.
3. $Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$.
4. $Z = 1.265$.
5. p-value $= 0.1029$.
6. Do not reject H_0.
7. The proportion of male births is 0.50.